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LOGICAL FORM*

Consider the following argument: All men are mortal; Socrates is a man; therefore, Socrates is mortal. Intuitively, what makes this a valid argument has nothing to do with Socrates, men, or mortality. Rather, each sentence in the argument exhibits a certain *logical form*, which, together with the forms of the other two, constitute a pattern that, of itself, guarantees the truth of the conclusion given the truth of the premises. More generally, then, the logical form of a sentence of natural language is what determines both its logical properties and its logical relations to other sentences.

The logical form of a sentence of natural language is typically represented in a theory of logical form by a well-formed formula in a 'logically pure' language whose only meaningful symbols are expressions with fixed, distinctly logical meanings (e.g., quantifiers). Thus, the logical forms of the sentences in the above argument would be represented in a theory based on pure predicate logic by the formulas '$\forall x(Fx \supset Gx)$', 'Fy', and 'Gy', respectively, where 'F', 'G', and 'y' are all free variables. The argument's intuitive validity is then explained in virtue of the fact that the logical forms of the premises formally entail the logical form of the conclusion. The primary goal of a theory of logical form is to explain as broad a range of such intuitive logical phenomena as possible in terms of the logical forms that it assigns to sentences of natural language.

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1 Logical form in Aristotelian logic
2 A general characterization of logical form
3 Comparing theories of logical form

1 Logical form in Aristotelian logic

Broadly speaking, the logical form of a sentence of natural language is what determines both its logical properties and its logical relations to other sentences. Most notably, the logical form of a sentence S determines whether or not it is logically true, and the logical

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* "Logical Form", in E. Craig (ed.,) *Routledge Encyclopedia of Philosophy*, vol. 5, London and New York: Routledge, 1998

forms of the sentences in a set K together with the logical form of S jointly determine whether or not the latter is a logical consequence of the former.

Although particularly prominent in the twentieth century, due especially to the influence of Bertand Russell, the idea of logical form can be traced back to Aristotle. In the so-called 'traditional logic' that stems from Aristotle's work in the *Prior Analytics*, the central form of argument is the *syllogism*, that is, an argument consisting of two premises and a conclusion, for example:

(1) Every whale is a mammal.
Some carnivore is a whale.
———————————
Some carnivore is a mammal.

Aristotle was the first to recognize explicitly that the validity of a syllogism (i.e., its conclusion being a logical consequence of its premises) is entirely independent of the common noun phrases, or *terms*, in its constituent sentences. Rather, each sentence in a valid syllogism like (1) exhibits a certain form which, combined with the forms of the other two, constitute a pattern that, of itself, guarantees the truth of the conclusion given the truth of the premises.

To capture this idea systematically, Aristotle first restricted his attention to syllogisms whose constituent sentences exhibit any of four basic sentential forms, known traditionally as A, E, I, and O, respectively: *Every a is a b*, *No a is a b*, *Some a is a b*, and *Some a is not a b*, where '*a*' and '*b*' represent the roles of the subject and predicate terms in such sentences. Thus, the first premise of (1) is of the sentential form A, and both the second premise and the conclusion are of the form I. These sentential forms alone, however, are not enough to characterize the pattern that (1) exhibits. A further logically relevant feature of (1) is the way in which the terms occur in its consitutent sentences: 'carnivore', for instance, is the subject term of both the second premise and the conclusion but does not occur at all in the first premise. To capture this feature generally, Aristotle introduced schematic variables *F*, *G*, *H*, ... Call the result of replacing the terms in a sentence that exhibits one of the basic sentential forms with distinct schematic variables a *schematic* form. The pattern exhibited by a given syllogism can then be represented by replacing its constituent sentences with schematic forms that preserve the arrangement of terms in the syllogism. (1), for instance, exhibits the pattern represented by (2).

(2) Every *F* is a *G*.
Some *H* is an *F*.
―――――――――
Some *H* is a *G*.

By means of representations like (2) alone, Aristotle proved, for each possible syllogistic pattern, whether or not it is valid, i.e., for Aristotle, whether or not it is impossible for the premises of any instance of the pattern to be true and the conclusion false. Thus, the schematic forms of its premises and conclusion (relative to a given choice of schematic variables) completely determine the validity or invalidity of each pattern. Aristotle's schematic forms are thus paradigms of logical forms.

The central goal of a *theory* of logical form is to explain the logical properties of (and relations among) the members of a broad class K of sentences of natural language in terms of the logical forms that the theory assigns to those sentences. With only four general types of logical form to choose from — the four basic sentential forms — the scope of Aristotle's account as it stands is far too limited to count as a full fledged theory of logical form. However, a perusal of any modern text containing a section on traditional logic illustrates a variety of techniques for *translating* sentences with entirely different grammatical forms into instances of the four basic sentential forms. For example, simple individual assertions like

(3) Matthew is a politician

are obviously not instances of any of the basic sentential forms, since they do not begin with one of the quantifiers 'Every', 'Some', and 'No'. However, by inventing a term that picks out a class containing only Matthew, (3) can be translated into the intuitively equivalent, albeit somewhat stilted, sentence

(4) Every person identical to Matthew is a politician,

which exhibits the sentential form A. Again, a sentence like

(5) Someone gave *The Brothers Karamazov* to Andrea

that involves a transitive verb can be cast into the appropriate sentential form by replacing the verb with its nominal –*er* counterpart followed by the preposition 'of', which produces a term that picks out exactly the class of individuals to which the original predicate applies. (5), then, gives way to

(6) Some person is a giver of *The Brothers Karamazov* to Andrea.

Supplemented by such techniques, the applicability of Aristotle's logic broadens considerably, as a much larger class of sentences can be assigned logical forms than initially appears. So supplemented, then, Aristotle's account is plausibly taken to be the first genuine theory of logical form.

2 A general characterization of logical form

Although differing considerably in content and detail, the basic components and characteristics of Aristotle's theory of logical form (in this perhaps somewhat anachronistic rendering) are essentially the same as in contemporary accounts deriving from Russell. In this section, these are made explicit.

In general, the notion of logical form is always relative to a theory T of logical form that is directed toward some broad class K of sentences of natural language. Call K the *target class* of the theory. The logical forms of the members of K are represented by formulas in a "logically pure" canonical language, i.e., a language that contains (perhaps in addition to punctuation) only variables and logical constants — expressions with fixed and, according to the theory, distinctly logical meanings (see LOGICAL CONSTANT). For Aristotle, this is the language of schematic forms whose logical constants are 'Every', 'Some', 'No', 'is a', and 'is not a'; and for Russell and his followers it is generally (some variant of) the language of *Principia Mathematica* with its now familiar quantifiers and propositional connectives. The route from the sentences of the target class K to their logical forms according to T consists of two steps. First, and most important, is the *translation* of the members of K into an 'impure' hybrid language that consists of the canonical language supplemented by nonlogical constants. These constants are drawn from nonlogical expressions occurring in the members of K: for Aristotle, both ordinary terms like 'carnivore' and stilted terms like 'person identical with Matthew'; for Russellians, individual constants like 'Socrates' and *n*-place predicates like 'mortal' (or perhaps abbreviations thereof like 's' and 'M'). The nonlogical constants carry the same meanings as their informal counterparts, and hence are used to construct the hybrid language translations of the sentences in which those counterparts occur. Thus, (3) and (5) are translated as (4) and (6), respectively, while for the Russellian, (7), for example, is translated as (8).

(7) If every man is mortal and Socrates is a man, then Socrates is mortal.

(8) $(\forall y(\text{man}(y) \supset \text{mortal}(y)) \ \& \ \text{man}(\text{Socrates})) \supset \text{mortal}(\text{Socrates})$.

The translation of a sentence of K into a hybrid language is typically known as its *analysis*. (Sentences like 'Every boy danced with a girl' that are logically ambiguous, of course, should receive a distinct analysis for each possible reading.) The analysis of a sentence is a paraphrase of the sentence that displays its logical form overtly in its surface grammatical structure. The logical forms *per se* for the sentences of K can be derived, then, simply by uniformly replacing all the nonlogical constants in the analyses of those sentences with variables of the appropriate types. So, for example (assuming a choice of replacement variables for the nonlogical constants), the sentences in (1) — which in this case are identical with their analyses — yield the logical forms in (2), and (dropping unnecessary parentheses) (8) likewise yields (9) as the logical form of (7).

(9) $(\forall y(Fy \supset Gy) \ \& \ Fx) \supset Gx$

Now, as noted, the primary goal of a theory T of logical form is to account for the logical properties of the sentences in its target class K in terms of their logical forms. This is accomplished by means of a logical theory for the canonical language of T in which formal correlates of the ordinary logical notions — logical truth, logical consequence, etc. — are defined for the formulas of the language. The apparent logical properties of the members of K are then explained (or explained away) in virtue of the fact that the formal correlates of those properties hold (or fail to hold) among the logical forms of the members of K. Thus, once again, the intuitive validity of (1) is explained by the formal validity of (2). Similarly, for the Russellian, the intuitive logical truth of (7) is explained by the formal logical truth of (9) — typically understood as truth in any domain under any interpretation (relative to that domain) of the free variables '*F*', '*G*', and '*x*', following the justly famous work of Tarski (see MODEL THEORY). In either case, the actual meanings of the nonlogical expressions in the natural language sentences in question play no role in determining the logical properties of those sentences. Rather, as the theories demonstrate, they are determined by their logical forms alone.

3 Comparing theories of logical form

The central principle for comparing theories T, T′ of logical form is that

CP T is preferable (*prima facie*) to T′ if T explains a greater range of logical behaviour than T′ without appeal to extralogical principles (sometimes known as 'meaning postulates').

To illustrate, intuitively, (10) follows directly from (5):

(10) Someone gave something to Andrea.

However, in Aristotle's theory, as noted, (5), as noted, must be analyzed along the lines of (6), and (10) along the lines of (11):

(11) Some person is a giver of something to Andrea.

But then (10) does not follow from (5) in virtue of their logical forms; rather, their analyses (6) and (11) exhibit only the obviously invalid argument pattern

(12) Some A is a B
―――――――――
Some A is a C.

To capture the original entailment, (12) must be treated as enthymematic: there is a suppressed premise with the logical form *Every B is a C*, namely,

(13) Every giver of *The Brothers Karamazov* to Andrea is a giver of something to Andrea.

Aristotle's logic, therefore, explains the relation between (5) and (10) by introducing an extralogical principle linking the two predicate terms, in effect, reducing it to an instance of the argument pattern (2).

By contrast, represented in predicate logic, the logical forms of (5) and (10) constitute the valid pattern

(14) $\exists x Hxyz$

$\overline{\exists y \exists x Hxyz.}$

Hence, that (10) is a consequence of (5) is explained on the basis of their logical forms alone without appeal to any extralogical principles. So, *prima facie*, a theory based on predicate logic is preferable to Aristotle's theory.

The '*prima facie*' qualification in CP marks the influence of other factors besides extent of explanatory range that are relevant to an evaluation of competing theories of logical form. Two particularly influential principles can be observed in recent debates over the logical forms of one or another class of sentences: *logical* conservativism and *ontological* conservativism.

The shift from Aristotle to Russell exhibits a typical way in which one theory T of logical form can broaden the explanatory range of another T′, namely, by extending or otherwise subsuming the canonical language and logic of T′. In such cases, the language of T is able to provide logical forms for all sentences in the target class of T′. In addition, however, the language of T includes logical constants and classes of complex expressions not present in the language of T′ which enable T to construct logical forms not available to T′. Appropriate logical principles for its new forms then enable T to explain logical phenomena that are left unexplained (without extralogical principles) in T′, for example, (10)'s being an logical consequence of (5). The principle of logical conservativism stipulates that supplementation to a given canonical language, should kept to a minimum. More exactly, let T and T′ be theories with roughly the same target classes, and let *L* be a background logical theory (first-order logic, for example) that is common to both T and T′; then

LC T is preferable (*prima facie*) to T′ if T requires fewer extensions to *L* than T′.

The increase in explanatory range of predicate logic, of course, is far too vast for LC to override CP in the choice between Aristotle and Russell. A better example is found in recent well known work on the logical form of action sentences with adverbial modifiers, for example,

(15) April kissed Jonathan tenderly.

It is intuitively clear that (15) entails

(16) April kissed Jonathan.

Typical predicate logic analyses of these sentences represent 'kissed' and 'kissed tenderly' as distinct two-place predicates. The entailment is then explained by means of a meaning postulate to the effect that if one person kisses another tenderly, then the former kisses the latter *simpliciter*. As with Aristotle's explanation of the relation between (5) and (10), then, on this explanation the apparent entailment is actually enthymematic. However, the entailment can be explained directly by treating an adverb like 'tenderly' as an adverbial operator t that, when prefixed to an n-place predicate F (like 'kissed'), yields a new n-place predicate $[tF]$ ('tenderly kissed'). The logic of these new constructions is then characterized generally by the principle that

(17) $[\alpha P]x_1...x_n \supset Px_1...x_n$, for any adverbial operator a and n-place predicate P.

The logical forms of (15) and (16), then, on this approach, are (18) and (19), respectively,

(18) $[tF]xy$

(19) Fxy

and so by the new logical principle (17), (16) follows from (15) directly in virtue of their logical forms.

Most philosophers would probably agree that this increase in explanatory range is significant enough to override LC and warrant the added apparatus. However, Davidson proposes a logically more conservative analysis that avoids the new apparatus. Specifically, for Davidson, the proper analysis of action sentences takes the structure of an action verb like 'kissed' to involve an implicit parameter for an *event*. Thus, (15) is to be analyzed as

(20) $\exists x$(kissed(April, Jonathan, x) & tender(x))

(read, roughly, as *There is an event x such that x is a kissing of Jonathan by April and x is tender*), and (16) as

(21) $\exists x$(kissed(April, Jonathan, x)).

The entailment from (15) to (16) is then explained by standard logical principles governing conjunction and the existential quantifier, and the apparently hasty introduction of new constructions and unfamiliar logical principles is avoided. Since their explanatory ranges are the same, then, CP provides no support for the adverbial operator account over Davidson's, and so, by LC, Davidson's account is to be preferred.

As Davidson's account illustrates, however, conservativism with regard to logic is often accompanied by liberalism with regard to ontology — in this case, the postulation of events. The *ontological commitments* of a theory T of logical form consist of the kinds of things that must exist if the analyses that T assigns to the sentences of its target class are to be meaningful. The third principle — ontological conservativism — is that such commitments are to be kept to a minimum, i.e., more exactly, where again **T** and **T**′ have roughly the same target classes,

OC T is preferable (*prima facie*) to T′ if T has fewer ontological commitments than T′.

Unlike LC, this principle favors the operator account of adverbial modification over Davidson's more ontologically permissive account. The choice between the two thus turns on one's preferred brand of conservativism.

LC tends to be overridden by CP when they conflict, as additional apparatus is usually viewed as a small price to pay for greater explanatory range. However, more is often at stake in conflicts between CP and OC, as theories of logical form that have great explanatory range often exact a high price in ontological commitment: possibilia are introduced to explain sentences involving modality, intensional entities to explain the logic of attitude verbs, and so on. One must either choose between the two, or offer a competing account with the same explanatory range but fewer ontological commitments. Since Russell first formulated his theory of descriptions to counter what he saw as Meinong's ontological excesses, the development of competing theories logical form has been, and largely remains, the primary tool for metaphysical discovery and the central approach to metaphysical debate in the twentieth century.

See also: ARISTOTLE, INTENSIONALITY, INTENSIONAL LOGIC, LOGIC, MODALITY, MODAL LOGIC, MONTAGUE GRAMMAR, ONTOLOGY, W.V.O. QUINE, BERTRAND RUSSELL, TARSKI.

References and further reading

* Aristotle. (c. 350s-330s BC) *Prior Analytics*, trans. and commentary R. Smith, Indianapolis: Hackett Publishing, 1989. (Referred to in §1. The primary source for Aristotle's formal logic, esp. I.1-7.)

* Davidson, D. (1967) 'The logical form of action sentences', in *The Logic of Decision and Action*, ed. N. Rescher, Pittsburgh: University of Pittsburgh Press, pp. 81-95; reprinted 1980 with replies to critics in D. Davidson, *Essays on Actions and Events*. Oxford: Clarendon Press, pp. 105-148; see esp. pp. 137-146. (The source of Davidson's account of action sentences with adverbial modifiers discussed in §3.)

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* Tarski, A. (1933) *Pojecie prawdy w jezykach nauk dedukcyjnych*, Warsaw; trans. by J. H. Woodger 1956 as 'On the Concept of Truth in Formalized Languages' and reprinted in J. Corcoran (ed.), *Logic, Semantics, Metamathematics*, 2nd edition, Indianapolis: Hackett Publishing Co., 1983, pp. 152-278. (Referred to in §2. The *locus classicus* of model theoretic semantics.)

* Whitehead, A. N. and B. Russell. (1910) *Principia Mathematica*, Cambridge: Cambridge University Press. (Referred to in §2; probably the most influential work in logic in the twentieth century; extremely technical.)

Wittgenstein, L. (1921) *Tractatus Logico-Philosophicus*, trans. D. F. Pears and B. McGuinness, London: Routledge & Kegan Paul, 1961. (A unique, highly influential

work by Russell's most famous student in which a very interesting, somewhat eccentric notion of logical form plays a prominent role.)